

A mathematical model for a verse of the holy Quran

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Abstract:

In the holy Quran, God has mentioned that good and bad deeds are accounted for. The punishment for deeds, such as homicide, is expressed. One of the verses, which this paper is based on, is the 32nd verse of the Al Ma'idah chapter of the holy Quran. For better understanding of the verses investigated, sciences other than interpretations and narrations can be adopted. In this paper investigations are made by comparing and analyzing verse concepts with other verses and narrations, and matching commentators' comments with mathematical basics. The purpose of these investigations is to open new horizons and express a possibility to match Quranic and narrative views with sciences such as mathematics. It is seen that some religious basics, i.e. deed calculations and some other issues mentioned in this investigation, can be expressed and discussed by mathematical relations and sets. The interval and domain of a set of individuals are determined in the mentioned verse, and reward and penalty quantities are expressed as a function of intentions.

Key words: Homicide, salvation, intention, sets, mathematical relations.

1. Introduction and problem statement

One of the topics which is widely regarded and also considered in the holy Quran and narrations are calculations of good and bad deeds in the afterlife. The issue of how deeds are accounted for in the afterlife has always been in the mind of Muslims. We use [2] in all parts of this paper.

One of the main discussions, which vastly have been considered in the holy Quran and narrations are regarding the humans' good and bad deeds in the afterlife.

There are some verses in the holy Quran which have expressed how deeds are determined and calculated, but these verses are not easily understood and several questions come in mind. These questions arise due to worldly and afterlife calculation differences.

The worldly calculations are made based on our understanding, but the afterlife calculations are made by God, which we are not able to understand. We are not able to understand the meaning of some verses related to calculations or other issues due to our simple ways of thinking.

As an example, in the 261st verse of the Al Baqarah chapter of the holy Quran, God says:

The likeness of those who spend their wealth in Allah's way is as the likeness of a grain which groweth seven ears, in every ear a hundred grains. Allah giveth increase manifold to whom He will. Allah is All-Embracing, All-Knowing.

As seen in the above mentioned verse, the way God accounts for spending in His way is not understandable for us. It can be said that, as this kind of calculations are not understandable for the average human, such verses needs to be explained and interpreted. In the interpretation of this verse in the book of Al-Mizan it is stated:

“God almighty gives even more than 700 grains to whomever He wants, because He is vast and there is nothing that can stop him and limit his grace and mercy.” Further, a narration of Imam Sadegh (PBUH) regarding this verse is expressed:

“If a faithful changes his was so that he does good, God increases his deeds by several times, every good deed is made 700 times more...”

One of the verses which raise questions on this topic is the 32nd verse of the Al Ma'idah chapter, in which God expresses that salvation of one human is equal to the salvation of all mankind and the homicide of an innocent and non-corrupt human is equal to homicide of all mankind.

Undoubtly, for any person reading this verse a question arises; how is it possible that murdering one person is equal to murdering everyone, and saving one person is equal to saving everyone?

This verse has been explained and interpreted in many interpretations. It has also been narrated several times. This shows that these questions have been addressed from the times of the Imams. These interpretations and narrations will be addressed later.

A question which arises is whether this kind of verses are explainable and understandable by other sciences than narration interpretation. E.g. is mathematics capable to do this? Can mathematics be adopted to express and model verses like this? To the best of our knowledge there are no research similar to the work that we have done. Therefore because our research is a novel one, there are not any significant references to be reviewed.

In this paper, the views of interpreters, different narrations, and questions and answers regarding this sacred verse will be expressed. Later a mathematical model for explaining this verse will be presented. First, the mathematical basis needed for explaining this verse using mathematics will be presented. Later, the relevant model will be explicitly presented.

2. Investigation Method

The investigation method adopted in this study is descriptive, analytical. Relevant information is first collected and then analyzed and discussed.

The main project hypothesis is based on the question whether Gods method for calculating deeds are comparable to our mathematical views. Thus, first the interpretations like the ones present in Al-Mizan, Majma-ol-Bayan and Al-Borhan are investigated. The interpreter opinions, regarding the 32nd verse of the Al Ma'idah chapter, are investigated. The opinions of pundits such as Allameh Tabatabaei on this regard are investigated. Of course, some other verses related to this issue are also investigated. Later, for justifying and explaining the related verse, interpretations and narrations are investigated and collected.

Further, as mathematics are used in this paper to present a new model, first some mathematical basics, and social explanations, simplifications, and illustrations, and some terms related to discussion as sets, relations, etc. are presented.

As the initial interpretation and narration views regarding the issue and also initial definitions of some mathematical terms are made, it is tried to match foundations expressed by religious jurists

and interpreters with mathematics. This is made by explaining how the murdering of an innocent non-corrupt person can be equal to murdering everyone and also how salvation of one person can be equal to the salvation of mankind. Then, the expressed opinions and Quranic and narrative opinions and views are compared and analyzed. It has also been attempted to validate the matching of verses and mathematical foundations by comparing and analyzing verses and narrations with the opinions of pundits.

3. Opinion expression and problem statement

Interpreters' opinions

Here, some interpretations of the 32nd verse of the Al Ma'idah chapter are investigated.

The 32nd verse of the Al Ma'idah chapter states:

On that account: We ordained for the Children of Israel that if any one slew a person - unless it be for murder or for spreading mischief in the land - it would be as if he slew the whole people: and if any one saved a life, it would be as if he saved the life of the whole people. Then although there came to them Our messengers with clear signs, yet, even after that, many of them continued to commit excesses in the land.

Regarding this verse the following is stated in Al-Mizan:

The word "Zaaleka" (this) is used in this verse as this story (the story of Cain and Able) is hinting at the nature of human. A nature which is not only limited to the sons of Adam, but any human pursuing their velleity is doomed to envy and hatred. The result of pursuing velleity will force him to dispute on the smallest thing others have. A question which arises in this verse is why this is stated to the sons of Israel, and whether this issue conflicts with Gods general addressing or not.

Allameh Tabatabaei responds:

This verse is revealed to the sons of Israel and there is no conflict regarding that this verse benefits both the sons of Israel and the general public. As other words of wisdom and preachings in the Holy Quran are for the Prophets' (PBUH) nation. It has this feature and its benefit is not

peculiar to the Prophets' (PBUH) nation, as for this verse. If the phrase "sons of Israel" is mentioned, it is due to that all investigated verses were for advising and warning them. As they envied, were pugnacious, and hostile towards the Prophet (PBUH), and fought the Muslims.

Citing and addressing the nation of sons of Israel can specify the amount of sin in a specific place and its importance. This issue will be regarded later.

The phrase "Man ghatala nafsan begheire nafsen" in this verse shows that God has excluded other homicides. E.g. a homicide which is performed as a nemesis, as God states in the Al Baqarah chapter, verse 179:

In the Law of Equality there is (saving of) Life to you

In this verse, it is stated as life and it is not seen as murdering all mankind.

Another homicide, which is excluded, is in the 178th verse of the Al Baqarah chapter. It is regarding murdering based on crime and corruption on earth, e.g. combatants and enemies of God. The murdering of these is not the same as murdering all mankind. Accidental kills, as different kinds of accidents or unintended murdering, or murders as in the story of prophet Khezr (PBUH) where the killing of a child is preferred to let him live with atrocities and enter the faction of the wicked, are other exclusions.

In the story of prophet Khezr (PBUH) and prophet Moses (PBUH), as stated in the Al Kahf chapter, verse 65 onwards, one of the accidents included the murder of an teenager by prophet Khezr (PBUH). The 74th verse of Al Kahf chapter:

Then they proceeded: until, when they met a young man, he slew him. Moses said: "Hast thou slain an innocent person who had slain none? Truly a foul (unheard of) thing hast thou done!"

The reason for this murder is stated in the 80th verse of the Al Kahf chapter:

As for the youth, his parents were people of Faith, and we feared that he would grieve them by obstinate rebellion and ingratitude (to God and man).

This killing was according to a divine plan and it was performed as God told prophet Khezr (PBUH). According to divine developments there are no obstacles for God to involve even a child in any incident, as the existence of this child might result in great dangers in the future.

The separating factor in the above cases is intention, i.e. the type of intention in the killing can result in different penalties and eventualities. In the investigated verse, only homicides in which a person is murdered without murdering another or inflicting corruption on earth are regarded. This type of murder is regarded as killing all mankind.

Regarding all humans, Allameh Tabatabaei says:

As a person carries on a truth and lives and dies with it. This truth is the same in everyone. There is no difference between one and all. Everyone are the same truth. The necessity for this definition is that murdering one person is like murdering all humans and vice versa, reviving one is like reviving all. This is the truth which the mentioned verse is hinting at.

It is possible to make the mistake that the 32nd verse of the Al Ma'idah chapter is a denunciation of purpose, as the main aim of this verse is to highlight the importance of homicide and the necessity of this aim is to express how increased number of homicides and the amount of crimes has a larger effect. Not that murdering one is the same as murdering all mankind. Another issue is that, if the murder of one is for example equal to murdering ten, then, according to the verse, the murdering of the first is equal to murdering all mankind, thus the murdering of the rest will be without any effect.

Another issue is that, increased sin and the punishment are incompatible with the Quran, as God in the 160th verse of the Al An'am chapter says:

He that doeth evil shall only be recompensed according to his evil

Another issue, how is it possible that a homicide can be equal to murdering all mankind. This requires that a set is equal to its own and other sets. Which is completely impossible, and if it is intended that a single homicide in a set is equal to murdering all but that set, it is as saying that "if someone murders an Iranian, it is equal to murdering all Japanese".

Of course, this is a very shallow and incorrect interpretation which wastes the main meaning and aim of this verse, which is to express the importance of oppression. Some have said that the meaning of this verse is to express the punishment or at least increased punishment, which this response is not sufficient for explaining the issue stated.

But the response to these issues can be seen in “man ghatal nafsa ...”, which quips that all humans are the same in one truth. So, if someone attempts to hurt someone, he tries to hurt everyone, as he tries to hurt humanity.

As water splits between numerous containers, if someone drinks the water in one of those containers, he empties that container of water and the other containers content does not change at all. Thus, as one drinks the content of one container he has drunk the content of all containers. The next sentence states: Va man ahyaha fakanama ..., the statement justifying it is the same as the interpretation of the previous sentence. The meaning of resurrecting is not to create a living human or resurrecting a dead person, but it is to give life as praxis.

In the translation of Kanz-al-Erfan the simile of the word “fakanam” in the above mentioned verse has several different narrations and opinions:

1. The purpose of the simile is to say that, one whom murders one unjust, is like one whom has murdered everyone. As others, due to his deed, will hate him, be hostile against him, and become his enemy.
2. It means that the result of that crime and sin is very heavy and grave, it is like he has murdered everyone.
3. From the look of the slain, it is as he has murdered everyone.
4. If he would murder everyone, how would he be killed? Now he should be killed and punished in the same way. It makes no difference for him.

In the translation of Majma' al-Bayan the description of this verse is as follows:

As a group of “fighters” believe, the meaning of this is that all humans become enemies with the slayer due to the catastrophic crime of manslaughter. The deed seems to be so that it seems the

slayer intends to slay everyone. As a result, everyone become like the victim and his relatives become distressed and sad, and they think of him as the enemy of mankind.

But, with regard to someone who saves someone from drowning, burning, dying beneath rubble, or any other life threatening danger, or guides someone to salvation, the reward of his work is as if he has given life to everyone. God rewards him as a revival. Anyway, such a person is great in people's minds. It seems he is preserving the respect of everyone as he is preserves the respect of one.

Ibn-Abbas believes that it is meant that if someone murders a prophet or a savior it is as if he has murdered everyone, and he will suffer the punishment of murdering everyone. If someone supports a prophet or just Imam, he will be rewarded as he has revived all mankind.

Another group, such as Ibn-Massoud, says that the meaning is that whoever murders someone unjust is regarded as he has killed everyone by the slain and his relatives. If someone who offers life is seen as he has given life to everyone by the person saved.

By reviewing different interpretations, it can be concluded that murdering an innocent and non-corrupt person is regarded as a great sin which is equivalent to the murdering of all mankind. Also saving a person who can revive someone, conceptually or

in any way, is so important and its reward is so vast that it is equivalent to save and revive all mankind.

3.1. Narrations, questions and answers related to the 32nd verse of the Al Ma'idah chapter

Different narrations regarding the 32nd verse of the Al Ma'idah chapter are available. Most of these are addressing questions regarding this verse.

- 1) Late Koleini narrates Hamran who once asked Imam Sadeq (PBUH) regarding the meaning of the 32nd verse of the Al Ma'idah chapter. I asked and said: "How is it that "Fakanam ghatala nas... َ" as he has only murdered one?", he said: "He is in a place in hell that the amount of torment for those there is that much, and if someone murders all

- mankind, he will be tormented in the same place.”, I said: “If he murders yet another?”, He said: “His torment will be twice as much.”
- 2) Late Koleini narrates Muhammad ibn Moslem, who narrates Imam Baqer (PBUH) regarding the verse “Man Ghatala ...”. I asked and he replied: “He is in the infernos of hell, it is the only place to be sent to if you murder all mankind.”
 - 3) Late Koleini narrates Samae’h who narrates Imam Baqer (PBUH): They asked him regarding the verse: “Man ghatala nasan begheire nasen ...”. I asked and he responded: “Whomever guides a lost soul has revived it and whomever makes a soul get lost has truly murdered it”.
 - 4) Sheykh Mofid also narrates Samae’h on the same narration: “I asked Imam Sadeq (PBUH): God revealed this verse “Man ghatala nasan begheire nasen” and the exact same narration is told and only at the end it differs as follows: “By God that he has murdered him.
 - 5) Late Koleini narrates Ibn Yasar that he asked Imam Baqer (PBUH) regarding the verse “^oVa man ahyahan fakanama nase jamian^o”, and he replied: “By blaze or drowning?”, I said: “And he who guides the lost?”, he responded: “This is the greatest interpretation of this verse.”
 - 6) Late Koleini narrates Hamran who stated: “I asked Imam Sadeq (PBUH): ”May I be sacrificed for you! may I ask?”, he replied: “Yes”, I said: “I feel different than before; As I entered a land and prayed for a man, a girl, a boy and a woman and God saved whoever He willed, but now I do not pray for anyone. Why am I so?”, he said: ”It is not your duty meddle between people and their God, as as God leads whoever He wants from the shadows to light.”, I said: “Tell me about the verse “^oVa man ahyahan fakanama nase jamian^o, he said: “It is meant by blaze or drowning.”. He became silent and then he said: “The greatest interpretation of this verse is to call onto the soul and respond to his soul.”“.
 - 7) Late Koleini narrates Ali ibn Ibrahim who narrates his father, Ibn abi A’mir, who narrates Moa’viah ibn A’mar, who narrates Imam Sadeq (PBUH) who has told: “He who gives

people water, where there is water, it is like a person who frees a prisoner, and he who gives people water in a place where there is no water, is like a person who gives life to a soul “^وVa man ahyahan fakanama nase jamian^ل ^و”.

- 8) A'yash narrates Hamran ibn Ay'in who speaks of Imam Sadeq (PBUH) regarding the verse “Man ajal zaleka katabna...” until the portion where it states’ “Fakanam ahyahan nasa^و”, I asked and he told: “In inferno there is a place where the acme of torment is inferno, he will be placed there.”, I said: “What if he murders two?”, he replied: “Can you not see that there is no place with more torment in hell?”, and said: “Then his torment will increase based on his deeds.”. I said: “And the one who revived the soul?”, he replied: “Meaning that he saved him from blaze, drowning, a predator or enemy.”, then he went silent and turned towards me and told: “The greatest interpretation is to call onto the soul and heed to the soul”.
- 9) Hanan ibn Sadir asked Imam Sadeq (PBUH) regarding the verse “Man ghatala nasan begheire nasan”, he replied: “There is a desert in hell in which you will be placed if you murder all mankind and you will be sent there if you murder even one”
- 10) Muhammad ibn Moslem has told of Imam Baqer (PBUH). He tells: “I asked him about the verse “ Man ghatala nasan begheire nasan.”, and he responded: “He has a place in inferno where he will be placed as if he has murdered all mankind, his torment will not be more than that”, and he told: “^وVa man ahyahan fakanama nase jamian meaning, he does not kill them, or save them from drowning or blaze and the greatest interpretation is to guide a lost one.”.
- 11) Ibn Basir tells of Imam Baqer (PBUH): “I asked him of “Men Ajle zaleka alaa bani esraiel””, he told me: “It means to change an unbeliever to a believer.”

In this section narrations and opinions on the mentioned verses were presented. It was seen that murdering one or all has the same torment, and murderers will be in one of the worst places of hell, where the amount of torment is acme. If someone would murder all mankind, he will be tormented in the same place. If he would murder two persons, his torment will be doubled.

4. Definition and introduction to relevant mathematics

As mathematical models are to be presented to express reward, penalty and the content of some verses, especially the 32nd verse of the Al Ma'idah chapter, and some related Quranic and narrative topics will be matched with mathematical basis, such as sets, relations, etc.

Sets and set operations

We use [1] to recall some definitions and preliminaries:

Set: A set is a well-defined collection of objects. The objects that make up a set are known as elements or members of a set. These can be anything: numbers, people, letters of the alphabet, other sets, etc.

Member: If A is a set and b is an element of A , it can be expressed as $b \in A$ and if c is not an element of A , it can be expressed as $c \notin A$.

Example: If 1, 2 and 3 are elements of a given set A , it can be expressed as $A = \{1, 2, 3\}$. This can also be expressed as: $1 \in A$, $2 \in A$ and $3 \in A$. It is clear that $4 \notin A$.

One set characteristic is that the equality of two sets is order independent. E.g. $\{1, 2, 3\}$ and $\{3, 2, 1\}$ two equal sets. If two sets A and B are not equal, it is written that $A \neq B$.

For example, the set $\{x\}$ has one element, x . The set $\{x, y\}$ has two elements, x and y . A set smaller than a single member set is called as an empty set, denoted as ϕ or $\{\}$, and it does not contain any element or member.

Subsets: The set A is a subset of B , if all elements of A are also elements of B . It is written that $A \subseteq B$. If A is not a subset of B it is written that $A \not\subseteq B$.

Example: If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$, it can be stated that $A \subseteq B$ and $B \not\subseteq A$.

Example: If $A=\{1,2,5\}$ and $B=\{2,3,6\}$, A and B are not each other's subsets. It can be written that $A \not\subset B$ and $B \not\subset A$ and each set is its own subset.

Note: The notations \in and \subseteq are different.

Example: If $A=\{1,2,3\}$ and $A_1=\{1\}$, then $A_1 \subseteq A$ and $A_1 \notin A$ and $1 \in A$

Example: If $A = \{a, \{a\}, b\}$, we will have the following:

- a) $\{a\} \in A$, $b \in A$, $a \in A$
- b) $\{a,b\} \subseteq A$, $\{\{a\},a\} \subseteq A$, $\{b\} \subseteq A$, $\{\{a\}\} \subseteq A$

Now all natural numbers are expressed as N and $N=\{1,2,3,\dots\}$.

Set determination: A method for determining a set is to determine the conditions for an element to be part of that set. I.e. an element is part of a set if it fulfills the conditions stated for that set. The set of all x elements, so that the condition x is needed to be part of that set is shown by the $\{x \mid \dots\}$ symbol, as shown below (x is the desired element and $'\mid'$ shows condition fulfillment):

$$\{x \mid \dots\}$$

Example: If N is the set of natural numbers.

- a) If $B_1 = \{x \mid x \in N, x > 7\}$, then $B_1=\{8,9,10,\dots\}$.
- b) If $B_2 = \{x \mid x \in N, x < 3\}$, then $B_2=\{1,2\}$.

The above examples are samples of choosing a set (the meaning of $> = <$ are smaller than, equal to, and larger than).

Set operations: Here two operations of union and intersection will be defined.

Union (U): Let's assume that A and B are two sets. The union of A and B is shown as $A \cup B$. The following are all elements of A and B :

$$A \cup B = \{x \mid x \in B \vee x \in A\}$$

Example: If $A=\{1,2,3\}$ and $B=\{2,4\}$, then $A \cup B = \{1,2,3,4\}$ and if O is a set of odd numbers and E is a set of even numbers, then $O \cup E$ is the set of all natural numbers.

$$N = O \cup E = \{1,3,5,7,\dots\} \cup \{2,4,6,8,\dots\}$$

Intersection (\cap): Let's assume that A and B are two sets. The intersection of these sets is shown as :

$$A \cap B = \{x \mid x \in B, x \in A\}$$

Example: If $A=\{1,2,3\}$ and $B=\{2,4\}$, then $A \cap B = \{2\}$ and if E is the set of all even and O is the set of all odd numbers, then $A \cap B = \phi$.

The power set of A ($P(A)$): If A is a set, then $P(A)$ is all subsets of A , called as the power set of A . This is defined as follows:

$$P(A) = \{X \mid X \subseteq A\}$$

Example: If A is a set of individuals in a house, consisting of Hassan (H), Reza (R) and Mahmoud (M), all possible combinations of these individual to be present in the house are as follows:

- 0- No one is at home (ϕ , an empty set), one possibility.
- 1- Only one person is at home (a single member set), three possibilities.
- 2- Two persons are at home (a two member set), three possibilities.
- 3- All three are at home (a three member set), one possibility.

Thus a set including all above possibilities is called a power set of A , $P(A)$. This power set is as follows:

$$P(A) = \{ \phi, \{R\}, \{H\}, \{M\}, \{H,R\}, \{M,R\}, \{M,H\}, \{M,H,R\} \}$$

Example: If $A=\{a,b\}$, then $P(A) = \{ \phi, \{a\}, \{b\}, \{a,b\} \}$.

One to one correspondence: There exists a one to one correspondence between A and B if each member of A has one and only one corresponding member in B , and vice versa.

Finite and infinite sets: A set which is empty or equivalent to a portion of natural numbers is called as a finite set. Any set which is not finite is called as an infinite set.

Relations: Relations, discussed next, are important concepts in theory of sets.

The concept of relations, are not only used in most mathematical branches, but it is also used in our everyday life. E.g when it is said that “Hassan is the brother of Hussein”, it is understood that they are related as “brothers”. When it is said “Muhammad is Mahdis’ father”, it is understood that Muhammad is related to Mahdi as a “father”. First ordered pairs and Cartesian multiplication of sets are defined.

Ordered pairs: Two sets $\{1,2\} = \{2,1\}$ are equal based on the definition of equal sets. The arrangement of the elements in the two sets does not affect the equality between the sets. Here, the pairs $(1,2)$ will be defined so that the arrangement of elements affect the equality, and thus $(1,2) \neq (2,1)$. The pair (x,y) will be defined so that:

$$x = u, y = v \leftrightarrow (x, y) = (u, v)$$

The pair (x,y) is called an ordered pair, and x and y are called as the first and second components, respectively.

Cartesian multiplication: Assume that A and B are two sets. The Cartesian multiplication of these sets is defined as follows:

$$A \times B = \{(a,b) \mid a \in A, b \in B\}$$

In other words, $A \times B$ consists of all ordered pairs (a, b) where the first component a belongs to the set A and the second component b belongs to the set B . Then both A and B are named as domains.

Example: If $A = \{a,b\}$ and $B = \{1,5\}$, then $A \times B = \{(a,1), (a,5), (b,1), (b,5)\}$.

Now if R is the relation of A to B has the domain of A and B ; ($R: A \rightarrow B$), then

$$R \subseteq A \times B.$$

Example: If A is a set with four elements, then the number of relations A can have in $P(A)$ is 2^4 (as $n(P(A)) = 2^{n(A)}$, thus, here $n(P(A)) = 2^4$).

The mathematical symbols \wedge , \vee , and \exists mean ‘and’, ‘or’, and ‘exists’, respectively.

Example: If $A = \{a, b\}$, as in the previous example, then $P(A) = \{\phi, \{a\}, \{b\}, A\}$, and then $R: A \rightarrow P(A)$ is a relation of A to $P(A)$. Assume R to be a relation:

- a) The scope or domain of R , $dom R$, is:

$$domR = \{x | \exists y((x, y) \in R)\}$$

In other words, the domain of R is the sum of every first component of the elements of R .

- b) The domain of values or range of R , $ran R$, is:

$$ranR = \{y | \exists x((x, y) \in R)\}$$

In other words, the range of R , is the sum of every second component of the elements of R .

Example: Below, the domain and range of each of the relations in a) and b) are specified:

a) $R_1 = \{(a, b), (1, 2), (1, a)\}$

b) $R_2 = \{(x, y) | x, y \in N, y = 2x\}$

a) and $ran R_1 = \{b, 2, a\}$

b) $domR = \{x | x \in N \wedge \exists y(y \in N \wedge y = 2x)\}$, which gives $dom R = N$.

As $2x \in N$ and $(x, 2x) \in R$ yield for $x \in N$, thus $x \in dom R$

If $ranR = \{y | y \in N \wedge \exists x(x \in N, y = 2x)\}$, then $ran R$ is the sum of all even natural numbers, i.e.

$$ranR = \{2x | x \in N\}$$

Relations: If A and B are two sets and $R \subseteq A \times B$, then R is called as a relation from A to B . In a special case, if $A = B$, $R \subseteq A \times A$, then R is called as a relation on A .

Assume A and B to be two sets and R to be a relation. If R is a relation of A on B , then $\text{ran } R \subseteq B$ and $\text{dom } R \subseteq A$. The opposite is also true, i.e. R is also a relation of B on A .

Merging relations: Assume S and R to be two relations. The merging of R with S , shown as RoS , is a relation defined as below:

$$RoS = \{(x, z) \mid \exists y((x, y) \in S, (y, z) \in R)\}$$

Example: If $R = \{(1,1), (1,2), (a,3)\}$ and $S = \{(1,6), (3,1)\}$, then $RoS = \{(3,1)\}$ and $SoR = \{(1,6)\}$.

5. Analyzing and matching mathematical foundations with interpretation and narration opinions

The effect of intention and its relation with deeds

In the Islamic belief, intention is one of the most important discussions on faith. Intention is accounted for in every apparent and non-apparent deed. Intention and cognition are seen as the source of every action, and de facto of that intention is the action taken.

In the 32nd verse of the Al Ma'idah chapter, discussed here, a point regarding homicide is expressed. It is expressed that any homicide is not equal to the murdering of all mankind, only where someone is murdered without having murdered anyone or spreading corruption on earth is as if he has murdered all mankind.

In this verse it is seen how intention and execution of an action is of importance to God. Or as the Prophet (PBUH) says: Action happens with intention as an intermediate and for the executioner it is as intended.

The measure of human deeds is based on their intention. Of course the determination of intention is not easy in this world, but in the afterlife intentions will be known and by those, deeds will be measured and evaluated.

Intentions are so important that, even if a human is not able to do what he intended, he will anyway be rewarded. As Imam Sadeq (PBUH) has told: God almighty has, for the sons of Adam, chosen that if anyone sets for doing something good, but is not able to perform, a good deed is registered for him, and bad deeds do not register as someone intends, but as he performs a sin is registered. At another place Imam Sadeq (PBUH) has told: As man, during day, intends to pray in the middle of the night, if he falls asleep, God will register its reward for him, and he writes his soul a rosary and set his sleep as charity). In the 225th verse of the Al Baqarah chapter, God says:

God will not call you to account for thoughtlessness in your oaths, but for the intention in your hearts; and He is Oft-Forgiving, Most Forbearing.

In the 32nd verse of the Al Ma'idah chapter homicide with the worst intent is mentioned and is regarded as murdering all mankind.

Thus if a human, as a member of all humans, commits a murder, he will create a relation between his deed and his punishment in the afterlife, with the intention in his mind. According to the verse, if someone murders someone, without him murdering someone else or spreading corruption on earth, it is as if he has murdered all mankind.

Now, the below defined mathematical relation of the set of all humans are taken to all of its subsets. Thus, with regard to that this deed is performed with the worst intent, God relates this deed to the largest domain, i.e. all mankind. In mathematical form, this issue can be expressed as follows.

The set A is assumed as the set of all mankind, and write (x is assumed as a specific individual):

$$A = \{\dots, \text{me}, \text{you}, x, y, z, \dots\}$$

The overall relation $R_i^y(x)$: The relation due to deeds and so that x performs on y .

$S_1(B)$: Homicide of all members of set B .

The relation $R_i^y(x)$ is only due to the murdering of innocent and non-corrupt y by x .

The power set of A , $P(A)$, is previously explained. By assuming all humans as the set A , the power set $P(A)$ is defined as below:

$$P(A) = \{ \phi, \dots, \{you\}, \{me\}, \{x\}, \{y\}, \{me \text{ and } you\}, \dots, A \}$$

For every $S_1(P(A))$ the following is true:

$$S_1(P(A)) = \{S_1(\phi), \dots, S_1(\{x\}), S_1(\{y\}), S_1(\{x, y\}), \dots, S_1(A)\}$$

$$R_1 : A \rightarrow S_1(P(A))$$

$$R_1^y(x) = S_1(A)$$

And thus, $R_1 \subseteq A \times S_1(P(A))$

For as x commits intentional homicide, according to the 32nd verse of the Al Ma'idah chapter, $(x, S_1(A)) \in R_1$.

In comparison to the first part of the verse, regarding homicide, the second part, regarding salvation, it can be concluded that God relates homicide to the largest member of the domain (the power set) as homicide is performed with the worst intent. Similarly, the amount in saving, without any reservations, conditions or beliefs, the largest amount is considered.

It can be concluded that this verse is a very clear example and sample of Gods' beneficence, mercy and extreme consideration without any reservation and condition.

Rewarding the faithful

With regard to the section of the verse, a question arises: If saving an individual by a faithful and upright person, with the best intentions, will the reward be the same as saving all mankind?

The 160th verse of the Al An'am chapter can be used in response to this question:

Va man jaa belhasanate

He that doeth good shall have ten times as much to his credit: He that doeth evil shall only be recompensed according to his evil: no wrong shall be done unto (any of) them.

In the translation of the interpretation of Al-Borhan, there is a narration of Zerareh: “I was sitting by Imam Sadeq (PBUH), a person asked him of the verse “man jaa” and whether this verse is regarding persons who are familiar with this matter (the succession of the Prophets household (PBUT)), or not. He replied: “This verse is only addressing the faithful””. alborhan

With regard to the narration about the 160th verse of the Al An’am chapter, the stated question can be responded. First, it seems as if a faithful performs a good deed like saving a person, he will be rewarded ten times more, i.e. as much as if he has saved ten persons. Apparently, this contradicts the 32nd verse of the Al Ma’idah chapter, which tells of saving one is the same as saving all.

If the 160th verse of the Al An’am chapter is used as basis, the reward for saving a person by a faithful is contradicting with the 32nd verse of the Al Ma’idah chapter. On the other hand, if the 32nd verse of the Al Ma’idah chapter is used as basis for comparison, it can be concluded that saving one person, by anyone without any precondition, is the same as saving all mankind, and according to the narration of Imam Sadeq (PBUH), this is ten times more for the faithful.

A narration of Imam Baqer (PBUH) in the book of Al-Mizan, with regard to the first part of the 32nd verse of the Al Ma’idah chapter regarding the torment of homicide confirms a tenfold reward for a faithful believer.

Late Koleini states himself in Kafi, where he narrates Hamran, who has said that, he told Imam Baqer (PBUH): “What does the words of almighty God in the 32nd verse of the Al Ma’idah chapter mean? And how can murdering one be the same as murdering all mankind?”, he replied: “It means that he will be placed in a place in hell, where torment is acme. It is a place where one will go if one murders all”, I said: “What if the murderer murders again?”, he said: “His torment shall increase”.

As, according to the above narration, the torment increases as someone murders two, it can be concluded that the reward of a faithful’s deed of salvation can according to the before mentioned narration of Imam Sadeq (PBUH) and the 160th verse of the Al An’am chapter increase (tenfold). This shows of God’s mercy and dignity towards his faithful believers.

Expressing mathematical relations for reducing deeds

Some factors which can reduce the torment of sins are regret, repentance, and natural disasters.

Some factors which can decrease the reward of good deeds are to boast and annoy.

Is someone commits a sin, and then repents and regrets, he will definitely be forgiven by God.

But if his sin or incorrect deed makes someone suffer, he has to compensate the person, or persons, who have suffered, in addition repenting, so that he finally can be forgiven by God and satisfy those who have suffered.

Imam Ali (PBUH) has told: “betobate samhas sayeat^o” (By repentance, your sins will be forgiven).

This can be expressed by mathematics as follows:

The reducing relation is shown by T , defined as below:

$$T : S_i(A_k) \rightarrow S_i(B_j) \quad \phi \subseteq B_j \subseteq A_k$$

$$1 \leq i \leq n$$

(B_j is a strict subset of A_k and B_j can be equal to ϕ .)

Now, homicide due to an accident is regarded. It is clear that, as this type of murder is not intended. This can be expressed as:

$S_5(B)$: The homicide of the set B , for which $n(B) = k$.

R_5 : The relation due to the accident.

$$R_5 : A \rightarrow S_5(P(A))$$

$$R_5 : \{\text{all mankind}\} \rightarrow \{S_5(\phi), S_5(\{x\}), S_5(\{y\}), \dots, S_5(\{x, y\}), \dots, S_5(A)\}$$

Thus, we have:

$$R_5^y = S_5(B)$$

Then, the effect of reducing deeds will be as follows:

$$A \xrightarrow{R_s} S_5(B) \xrightarrow{T} S_B(D)$$

$$\text{Rang} R_s \subseteq \text{dom} T$$

and thus

$$\text{To} R_s^y(x) = \{S_5(B) \mid D \subset B, \phi \subseteq D \subset B, R_s^y(x) = S_5(B)\}$$

Some sins (related to suffering of persons) are forgivable, but some cannot fully be forgiven due to the different conditions these are having. Meaning that this kind of sins have to be forgiven by God and the person(s) who have suffered.

Thus, if someone has committed a sin, but is unable to satisfy a person who has suffered, after he has repented and asked for forgiveness from God, he cannot be fully forgiven and a penalty awaits him. E.g. for homicide, even if the murderer asks for Gods forgiveness, he cannot satisfy the victim, even after he is convicted. It is clear that after paying blood money and nemesis, some of his sin will be forgotten.

In the 93rd verse of the Al Nisa' chapter:

“Man yaghtelo momenan motaamedan...”

If a man kills a believer intentionally, his recompense is Hell, to abide therein (Forever):
And the wrath and the curse of God are upon him, and a dreadful penalty is prepared for him.

From this verse it can be deduced that eternal damnation awaits whoever murders a faithful believer intentionally. He will never be forgiven by God, nevertheless, he will be subjected to Gods' torment and wrath.

Imam Sadeq (PBUH) has, on this matter, said: “enaho ghaleho aala dine va...” «” (As he was a faithful believer and lost his life due to his belief and faith, there is no doubt or uncertainty that

such a deed is an indication of the murderers unbelief and will result in his eternal eternity in the fires of hell).

6. Results and proposals

The below results can be concluded:

- 1- Every person, performing any deed, creates a mathematical relation based on the intention which he has. Every person, as a member of the set of all humans, can create a relation by performing a deed, which affects a single member, two member, ..., or all members of the set. In the 32nd verse of the Al Ma'idah chapter it is stated that, if a single person, as a member of the set of all humans, murders an innocent person, a relation is created which finally, according to this verse, makes him equal to the murderer of the set of all humans. Regarding the salvation of one human the same is stated. Thus, the meaning of the mentioned verse is understandable and clear by mathematical means:

$$A = \{ \dots, x, y, k, z, you, I, \dots \}$$

In the state $R_1^y(x)$: It is a relation based on deeds and so that its subject is x and its object is y .

$S_1(B)$: The homicide of all members of a set.

The relation $R_1^y(x)$ is due to the homicide of innocent and non-corrupt y by x .

The power set of $P(A)$ is as below:

$$P(A)$$

$$= \{A \text{ and } \dots \text{ and } \{x, y\} \text{ and } \{you \text{ and } I\} \text{ and } \{x\} \text{ and } \{y\} \text{ and } \{you\} \text{ and } \{I\} \text{ and } \dots \text{ and } \phi\}$$

For $S_1(P(A))$ the following is true,

$$S_1(P(A)) = \{S_1(\phi), \dots, S_1(\{x\}), S_1(\{y\}), S_1(\{x, y\}), \dots, S_1(A)\}$$

$$R_1 : A \rightarrow S_1(P(A))$$

$$R_1^y(x) = S_1(A)$$

And thus, $R_1 \subseteq A \times S_1(P(A))$

As x commits intentional homicide we will have a ordered pair $(x, S_1(A)) \in R_1$, according to the 32nd verse of the Al Ma'idah chapter.

Another example of homicide is nemesis. No torment is regarded for this type of homicide, and according to the Al Baqarah chapter, verse 179, this is even regarded as a type of life.

Thus, based on the mathematical relation the penalty of nemesis will be void.

Repent is a penalty reducing relation. This relation mixes with the performed deed relation, and relates to a subset of the strict set.

- 2- The 32nd verse of the Al Ma'idah chapter is revealed addressing the Israeli nation. According to the discussions, it can be concluded that this verse is generalized to all mankind, and not only the Israeli nation. It is seen that, for good deeds, such as saving people, great reward await from God, even for a nation like the Israeli nation. As this reward is expressed for an oppressing nation, it has to be the least reward and mercy from God onto his servants. If such a deed is performed by a faithful believer, this reward will be tenfold, according to the 160th verse of the Al An'am chapter. Similarly, for the homicide of an innocent person, this is the greatest torment.
- 3- According to interpretations and narrations, related to the 32nd verse of the Al Ma'idah chapter, homicide and salvation are not only regarded by their apparent meanings. But, homicide and salvation are defined as misguidance and guidance, respectively. It can be said that every deed, related to wasting people's rights, property, etc., can be accounted as homicide of the soul (in a smaller scale) and any good deed related to people's rights can be seen as saving someone's life (in a smaller scale).

In this paper, mathematical relations are used for defining amounts for deeds. Questions which arise, which have to be discussed and studied in the future, are: Can functions be regarded as relations? What deeds can, what deeds cannot, be regarded as functions? Can fuzzy sets be proposed, for expressing more details? What will their effect can be? Can other deeds be related with homicide and salvation? Can deed and intention be separated? Can humans obtain criteria for evaluating rewards and torments?

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